CEE Growth & Development

Solving Solow Model

Michælmas 2013

World Development Report 2014

- Schebeck Palace Room 8

Example

Suppose that a constant 20 per cent of output is invested and capital stock depreciates at a constant rate of 0.8 per cent and population is constant. If the economy exhibits a Cobb-Douglas production function, with $\alpha = 1/3$ and $A = 1$, and the current level of capital per worker is 125, what will happen to capital stock?

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Solution

$$\begin{aligned}\Delta K &= s \cdot Y - \delta \cdot K \\ &= s \cdot A K_t^{\alpha} L^{1-\alpha} - \delta \cdot K\end{aligned}$$

$$\begin{aligned}\Delta k &= s \cdot A k_t^{\alpha} - \delta \cdot k \\ &= 0.2 \cdot 1 \cdot (125)^{1/3} - 0.008 \cdot 125\end{aligned}$$

Log and growth rate

- $$\frac{d}{dt}\log x = \left(\frac{d}{dx}\log x\right)\cdot\left(\frac{d}{dt}x\right) = \frac{\frac{d}{dt}x}{x} = \frac{\dot{x}}{x}\,[= g_x]$$

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- $$\begin{aligned} y &= k^\alpha \\ \log y &= \alpha \log k \\ \frac{\dot{y}}{y} &= \alpha\frac{\dot{k}}{k} \\ g_y &= \alpha \cdot g_k \end{aligned}$$

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- $$k = \frac{K}{L}$$
- $$\begin{aligned} \log k &= \log K - \log L \\ \frac{\dot{k}}{k} &= \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \\ g_k &= g_K - g_L \end{aligned}$$

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$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$$

$$\boxed{g_k = g_K - g_L}$$

Changes in Capital

Capital Widening vs. Capital Deepening

$$\frac{\dot{K}}{K} = \frac{\dot{k}}{k} + \frac{\dot{L}}{L}$$

Solow Model: Basics

Assumption 1. Production Function

$$Y = AK^{\alpha}L^{1-\alpha}$$

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- Parameters, Exogenous Variables, and Endogeneous Variables

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3. Easier

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta$$

$$\frac{\dot{k}}{k} + \frac{\dot{L}}{L} = s\frac{Y/L}{K/L} - \delta$$

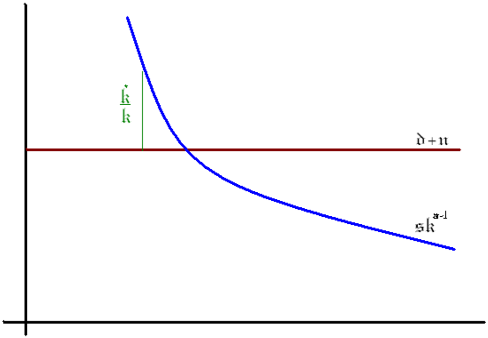
Solow Model: Transformations

1. Production Function
2. Fundamental Law of Motion
3. Fundamental LoM Easier

$$
\begin{aligned}
\frac{\dot{K}}{K} &= s\frac{Y}{K} - \delta \\
\frac{\dot{k}}{k} + \frac{\dot{L}}{L} &= s\frac{Y/L}{K/L} - \delta \\
\frac{\dot{k}}{k} + n &= s\frac{y}{k} - \delta \\
\dot{k} &= sAk^{\alpha} - (\delta + n)\,k
\end{aligned}
$$

Solow Model: Transformations

$$\frac{\dot{k}}{k} = sk^{\alpha-1} - (\delta + n)$$



Solow Model: Solving for SS

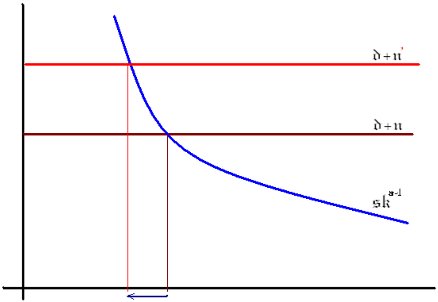
$$\left.\frac{\dot{k}}{k}\right|_{SS} = 0$$

$$sk^{\alpha-1} - (\delta + n) = 0$$

$$k|_{SS} = \left(\frac{s}{\delta + n}\right)^{\frac{1}{1-\alpha}}$$

Capital Dilution

$$k_{SS} = \left(\frac{s}{\delta + n}\right)^{\frac{1}{1-\alpha}}$$



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- Countries will eventually end up in their steady states
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- In their steady states the aggregates will grow (with the rate of population growth) while the per capita variables are constant
- The larger the population growth rate, the smaller the steady-state values of per capita variables are:

$$\frac{y_i^{ss}}{y_j^{ss}} = \left(\frac{n_j + \delta}{n_i + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$